1 Introduction
Fluorescence Molecular Tomography (FMT) is commonly used in small animals, most commonly mice. A significant challenge in imaging larger anatomies is the computation time required to perform both the forward model for light propagation and the matrix inversion for computing the distribution of fluorophores in the animal. We present a technique for reducing the size of the problem through the use of hybrid computations in real space and in Fourier space, which results in a speed-up in computation time by several orders of magnitude.

2 Forward Model
FMT reconstructions use a forward model for light propagation through tissue based on the commonly used diffusion approximation. This approximation uses Green’s functions to model the propagation of light between a grid of excitation light source locations, the voxels within the reconstruction, and the camera detectors. Accounting for absorption and scattering at the tissue-air interface as part of this model is computationally very intensive. The "Boundary Removal" technique described by Ripoll & Ntziachristos (2006) simplifies this computation by integrating the model with the scattering or absorption in real space. The voxel values are calculated by integrating the contribution of each source point. The result is a large number of voxels, which are kept in real space. This approach also has the advantage of avoiding the reconstruction artifacts and non-quantitative results that are obtained from a pure Fourier approach such as back-projection or direct inversion techniques. Furthermore, the direct inversion approach requires 10^13 source locations (Markel & Schotland, 2001), which is not practical for in vivo imaging, whereas the hybrid approach requires only 10^10 or fewer.

3 Optimized Frequency Components
A critical step in the development of this technique is optimizing the frequency of the weight matrix to strike an appropriate balance between the resolution of the final reconstruction and the computation time. The optimal resolution can be determined directly from the FWHM of a point source at the source-detector separation, which is translated into Fourier space. The FWHM is defined as 2π times the spatial frequency at which the intensity of the output is reduced to one-half of its maximum value. The FWHM in real space is given by:

\[ \Delta f = \frac{1}{2 \pi L} + \frac{\log(2)}{2 \pi L} + \frac{1}{2 \pi L} \]

From this, we use the relationship between the FWHM of a function and that of its Fourier Transform to derive a cut-off frequency in the range of 10^3 to 10^4. The FWHM in Fourier space is given by:

\[ K_{max} = 4 \pi \left( \frac{1}{2 \pi L} + \frac{\log(2)}{2 \pi L} + \frac{1}{2 \pi L} \right)^{1/2} \]

This limits the size of the weight matrix discussed above to N_x × N_y × N_z elements, where M stands for the number of the voxels being reconstructed.

4 Validation
Critically, this technique maintains the quantitative nature of real-space FMT reconstructions and accurate depth recovery, both in phantoms and in vivo. Sample in vivo results are shown below. These reconstructions showed good agreement both quantitatively and qualitatively with pure real space reconstructions of the same data. Increasing the value of K for these reconstructions had a significant effect on the apparent resolution of the reconstruction.

5 Summary
We have demonstrated a novel approach to reconstruction of fluorescence signal in a turbid medium using a hybrid forward model that transforms the detector data into Fourier space. By using a small number of frequency components, we reduce the size of the weight matrix by roughly 100-fold as compared to a purely real space approach. This improves computation times by well more than 100x. Maintaining the voxels in real space, as opposed to a pure Fourier approach, enables the method to avoid the reconstruction artifacts and non-quantitative results inherent in such techniques.

References
- Ripoll & Ntziachristos, 2006, PRL 96, 173903
- Markel & Schotland, 2001, PRE 64, 035601